# SIMULATION OF PARABOLIC-TROUGH COLLECTOR (MODEL TE38) USING SOLAR POSITIONS COORDINATES OF BAUCHI 

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#### Abstract

This paper presents the simulation of parabolic-trough collector (model TE38) using solar positions coordinates of Bauchi. The model equations included the universal time (UT), day ( $n$ ), month (M), year (Y), delta $T(\Delta T)$, longitude ( $\gamma$ ) and latitude ( $\phi$ ) in radian. The heliocentric longitude $(\mathrm{H})$, geocentric global coordinates and local topocentric sun coordinates were considered in the modeling equations. The thermal efficiency $\eta_{\text {th }}$ of the PTC considered both the direct $\left(\mathrm{E}_{\mathrm{gd}}\right)$ and reflected ( $\mathrm{E}_{\mathrm{gr}}$ ) solar energy incident on the glasscover as well as the thermal properties of the collector and the total energy losses ( $\mathrm{Q}_{\text {losses }}$ ) in the system. The model equations were used to simulate the PTC (Model TE38) using meteorological and radiative data of Bauchi. The results show that the temperature of the glasscover ( Tg ), absorber-tube ( Tt ), and the working fluid ( Tf ) increases with increase in time from 350C, 610C and 39oC to 43oC, 1250 C and 1170 C respectively. The efficiency of the PTC increases with increase in time to a maximum value of $95 \%$ at $11: 00 \mathrm{hrs}$ and reduces gradually to a minimum value of $62 \%$ at 19:00hrs.


Key words: Parabolic-trough Collector, Energy, solar coordinates, thermal efficiency, meteorological and radiative.

### 1.0 INTRODUCTION

### 1.1 Background of the study

Parabolic-trough Collector (PTC) consists of a reflecting surface made by bending highly polished stainless steel reflector and fixed on a parabolic contour. The reflecting surface may also be made of polished mirror. The parabolic contour is supported by steel framework and mounted on a reflector support structure. A hand-wheel operated tilting mechanism permits adjustment of declination, in order to track the sun, thus, maintains the focusing of the solar radiation on the receiver. The receiver assembly comprises of the absorber-tube covered with a glass-cover tube to reduce heat losses, is placed along the focal line of the receiver as shown in figure 1.

The definition of a parabola provides that all paraxial rays striking the parabolic reflector are reflected toward the line-focus. Concentration is achieved by using the reflector to channel natural concentration of energy on the reflector's aperture area into a significantly smaller area, the receiver assembly that is mounted on the focal line of the parabola. Typically, concentrator systems will not work efficiently without tracking, so at least single-
axis tracking is required. This is as a result of the continuous changing sun's position in the sky with respect to time of the day as the sun moves across the sky. Thus, the location of the sun may be predicted for any latitude, time of day, and any day of the year, using simple trigonometric formulae.

The accuracy of the solar tracking depends to a great extent on the solar coordinates at a particular location and at a given instant. This would greatly influence the solar radiation collection and thus the efficiency of the system. This necessitates the need to simulate the PTC using solar position coordinates in order to enhance the performance of the collector.

When the sun is observed from a position on the earth, the point of interest is to know the position of the sun relative to a coordinate system that is located at the point of observation not at the center of the earth. The position of the sun relative to these coordinates can be fixed by two angles; solar altitude angle ( $\alpha$ ) and the solar azimuth angle (A). These are as shown in figure 2 [1].

Williams and Raymond, [1] stated that solar altitude angle $(\alpha)$ at a point on the earth is the angle between the line passing through the point and the sun and the line passing through the point tangent to the earth and passing below the sun.


Fig. 2. Composite View of Parallel Sun Rays Relative to the Earth Surface and the Earth Center Coordinates

The solar azimuth angle (A) is the angle between the line under the sun and the local meridian pointing to the equator, or due south in the northern hemisphere. It is positive when measured to the east and negative when measured to the west. The solar zenith angle ( $\theta_{z}$ ) is the angle between a solar ray and local vertical direction and it is the complement of $(\alpha)$.

The time of the year is specified by the topocentric solar declination $\left(\delta_{t}\right)$. The time of the day is specified by the hour angle $\left(h_{t}\right)$. The hour angle is defined as zero at local solar noon and increases by $15^{\circ}$ for each hour. Solar time is location dependent and is generally different from local time, which is defined by time zones and other approximation. Some situations such as performance correlations, determination of true south, and tracking algorithms require an accurate knowledge of the difference between solar time and the local time [2].

Other critical factors to be considered in the analysis of the solar concentrating collector are its material properties. When radiation strikes a body, part of it is reflected, a part is absorbed and if the material is transparent, a part is transmitted. The fraction of the incident radiation reflected is the reflectance ( $\rho$ ), the fraction of the incident radiation transmitted is the transmittance $(\tau)$ and the fraction absorbed is the absorptance $(\alpha)$ as shown schematically in figure 3.


Fig. 3. Reflection, Absorption and Transmission of incident solar radiation by a solid body.

Absorption of solar radiation is what solar heat collector is all about; therefore, a blackbody surface is ideal surface for making any solar collector's absorbing surface. For this reason, the latter is usually made as black as possible so that it will have a high absorption capacity.

### 2.0 DEVELOPMENT OF SOLAR PARABOLICTROUGH ENERGY EQUATIONS USING SOLAR COORDINATES

In developing the model equations for computing the efficiency of PTC using solar coordinates, the input parameters are determined by the time scales because of the importance of using the correct time in the calculation of solar position for solar radiation application.

### 2.1 Determination of Delta $T(\Delta T)$

Delta $T(\Delta T)$ is a measure of the difference between a time scale based on the rotation of the Earth (Universal time UT) and an idealized uniform time scale at the surface of the Earth (Terrestrial time TT). The values of $\Delta \mathrm{T}$ can be obtained from the (TT - UT) smoothed data provided in Bulletin B of the International Earth Rotation and Reference Systems, (IERRS) [3]. The value of $\Delta T$ was obtained for the year 2014 to be approximately 67.7 seconds.

The Universal Time (UT), or Greenwich civil time, is used to calculate the solar position based on the earth's rotation and is measured in hours from 0-hour at midnight. The minutes and seconds must be converted into fraction of an hour.

### 2.3 Determination of Time Scale ( $t_{G}$ and t )

The time scale, $\left(t_{G}\right.$ and $t_{\text {) }}$ ) are the Julian day and the Ephemeris Julian day respectively. These are used to overcome the drawbacks of irregularly fluctuating mean solar time due to elliptical shape of the earth's orbit i.e. the interval between two successive passages of the sun through the meridian, is not constant. They are defined as follows [4]. $t_{G}=I N T(365.25(Y-2000))+I N T(30.6001(M+$

$$
\text { 1)) }+n+\frac{U T}{24}-1158.5
$$

Eqn. 1
$t=t_{G}+\frac{\Delta T}{86400}$
Eqn. 2
where $I N T$ is the integer of the calculated terms, $Y$ is the year, $M$ is the month of the year, n is the day of the month with decimal time and UT is the Universal Time.

### 2.4 Longitude $\boldsymbol{\gamma}$ and Latitude $\boldsymbol{\phi}$

Sun position is also very location-dependent, so it is critical that the longitude and latitude of the site are known before calculations are carried out. In this case the longitude and latitude of Bauchi are $9.81^{\circ} \mathrm{E}$ and $10.33^{\circ} \mathrm{N}$ respectively [5].

### 2.5 Determination of Heliocentric Longitude of the Earth

According to Reda and Andreas, [4] "Heliocentric" means that the earth position is calculated with respect to the
center of the sun. This is the most critical point, and the main source of errors. The heliocentric longitude (H) is the sum of four terms defined as follows;

$$
H=L_{y}+L_{m}+L_{h}+L_{p} \quad \text { Eqn. } 3
$$

where: $\quad \mathrm{L}_{\mathrm{y}}=1.74094+1.7202768683 \mathrm{e}^{-2 \mathrm{t}}+$

$$
3.34118 \mathrm{e}^{-2} \sin \sigma+3.488 \mathrm{e}^{-4} \times \sin 2 \mathrm{~A}_{\mathrm{o}}
$$

Eqn. 4
$A_{o}=1.72019 e^{-2 t}-0.0563$
Eqn. 5
$L_{m}=3.13 \mathrm{e}^{-5} \times \sin (0.2127730 \mathrm{t}-0.585)$ Eqn. 6
$\mathrm{L}_{\mathrm{h}}=1.26 \mathrm{e}^{-5} \times \sin \left(4.243 \mathrm{e}^{-3 \mathrm{t}}+1.46\right)+2.35 \mathrm{e}^{-5} \times$
$\sin \left(1.0727 \mathrm{e}^{-2 \mathrm{t}}+0.72\right)+2.76 \mathrm{e}^{-5} \times \sin \left(1.5799 \mathrm{e}^{-2 \mathrm{t}}+\right.$
$2.35)+2.75 \mathrm{e}^{-5} \times \sin \left(2.1551 \mathrm{e}^{-2 \mathrm{t}}-1.98\right)+$
$1.26 \mathrm{e}^{-5} \times \sin \left(3.1490 \mathrm{e}^{-2 \mathrm{t}}-0.80\right) \quad$ Eqn. 7
$L_{p}=\left(\left(\left(-2.30796 e^{-7} t_{2}+3.7976 e^{-6}\right) t_{2}-2.0458 e^{-5}\right) t_{2}\right.$
$\left.+3.976 e^{-5}\right) t_{2}^{2}$
Eqn. 8
Eqn. 9
where $L_{y}, L_{m}, L_{h}$, and $L_{p}$ are linear increase with annual oscillation, moon perturbation, harmonic correction and polynomial correction respectively. The time scale $t_{2}$ is introduced in order to have more homogenous quantities in the products within the polynomial, avoiding too rough rounding approximation [6].

### 2.4 Correction to Geocentric Longitude Due to Nutation ( $\Delta \boldsymbol{\gamma}$ )

"Geocentric" means that the sun position is calculated with respect to the earth center [4]. One problem is that the sun's apparent diurnal and annual motions are not completely regular, due to the ellipticity of the Earth's orbit and its continuous disturbance by the moon and planets. Consequently, the complex interactions between the bodies produce a wobbly motion of the earth rather
than completely smooth motion, thus the need to compute the correction to geocentric longitude defined by the following relation [6].
$\Delta \gamma=8.33 e^{-5} \times \sin \left(9.252 e^{-4 t}-1.173\right)$
Eqn. 10

### 2.5 Determination of the Earth Axis Inclination ( $\epsilon$ )

This is the inclination of the center of the earth axis to the sun and is correlated as follows [6].
$\varepsilon=-6.21 \mathrm{e}^{-9 \mathrm{t}}+0.409086+4.46 \mathrm{e}^{-5} \times \sin \left(9.262 \mathrm{e}^{-4 \mathrm{t}}+\right.$ 0.397)

Eqn. 11

### 2.6 Determination of Geocentric Global Solar Coordinates

This is the coordinates of the sun with respect to the earth's center.

### 2.6.1 Computation of geocentric solar longitude $\left(\gamma_{g}\right)$

The geocentric solar longitude may be estimated using the following relation [6].
$\gamma_{g}=H+\pi+\Delta \gamma-9.932 e^{-5}$
Eqn. 12

### 2.6.2 Computation of geocentric right ascension $\left(\alpha_{a}\right)$

The ascension angle ( $\alpha_{a}$ ) is defined by [6].
$\alpha_{a}=\arctan \left(\sin \gamma_{g} \cos \varepsilon \cos \gamma_{g}\right)$
where arctan is an arctangent function that is applied to maintain the correct quadrant of the $\alpha$ where $\alpha$ is in the range from $-\pi$ to $\pi$.

### 2.6.3 Computation of geocentric declination $(\delta)$

The geocentric solar declination is the declination of the sun with respect to the earth's center. The angle between the earth's equatorial plane and the earth-sun line varies between $\pm 23.45^{\circ}$ throughout the year. This angle is called the declination $(\delta)$. At the time of the year when the northern part of the earth's rotational axis is inclined toward the sun, the earth's equatorial plane is inclined
$23.45^{\circ}$ to the earth-sun line. At this time, the solar noon is at its highest point in the sky and the declination angle $(\delta)= \pm 23.45^{\circ}$ as illustrated in figure 4. This condition is called the summer solstice [7]. Thus, the declination can be computed at any given instant by equation 14.
$\delta=\arcsin \left(\sin \varepsilon \sin \gamma_{g}\right)$
Eqn. 14


Fig. 4. Solar Declination Angle

### 2.7 Computation of Local Hour Angle of the Sun (h)

The hour angle is the angle through which the earth must turn to bring the meridian of a point directly in line with the sun's rays. At solar noon the hour angle is 0 and it expresses the time of a day with respect to solar noon. It is measured positively west ward from the observer. One hour is equivalent to $\frac{2 \pi}{24}=0.262 \mathrm{rad}$ or $\frac{360^{\circ}}{24}=15^{\circ}[4]$. $h=6.3003880990 t_{G}+4.8824623+0.9174 \Delta \gamma+\gamma-$ $\alpha_{a}$

Eqn. 15

### 2.8 Computation of Parallax Correction to Right

## Ascension ( $\Delta \alpha$ )

The parallax correction may be computed using the following correlation [6].

$$
\Delta \alpha=-4.26 e^{-5} \times \cos \phi \sinh
$$

Eqn. 16

According to Reda and Andreas, [4] topocentric sun coordinates means that the sun position is calculated with respect to the observer local position at the earth surface.

The topocentric coordinates differ from the geocentric because they have their origin on the earth surface, and not on the earth center; the difference must be considered. Grena [6] stated that topocentric sun coordinates affects the sun positions by many seconds of arc.

### 2.9.1 Topocentric right ascension $\left(\alpha_{t}\right)$

The topocentric ascension angle is correlated as [6].
$\alpha_{t}=\alpha_{a}+\Delta \alpha$
Eqn. 17

### 2.9.2 Topocentric solar declination $\left(\delta_{t}\right)$

This is the declination as expressed in section 2.6 .3 with
respect to the local observer position. This angle is given as [6].
$\delta_{t}=\delta-4.26 e^{-5} \times(\sin \phi-\delta \cos \phi) \quad$ Eqn. 18
where $\delta$ geocentric declination and $\phi$ observer local
latitude.

### 2.9.3 Topocentric hour angle

This is as explained in section 2.7 and is determined with respect to the local observer position [6].
$h_{t}=h-\Delta \alpha$
Eqn. 19
$c h_{t}=\cosh +\Delta \alpha \sinh \left(\right.$ Appro.cosine of $\left.h_{t}\right)$ Eqn. 20
$s h_{t}=\sinh +\Delta \alpha \cosh \left(\right.$ Appro. sine of $\left.\mathrm{h}_{\mathrm{t}}\right)$ Eqn. 21

### 2.10 Determination of Solar Elevation Angle, Without

Refraction Correction ( $e_{o}$ )
Atmospheric refraction is the deviation of light or other electromagnetic wave from a straight line as it passes through the atmosphere due to the variation in air density as a function of altitude and is given by [8].
$e_{o}=a \sin \left(\sin \phi \sin \delta_{t}+\cos \phi \cos \delta_{t} c h_{t}\right)$
Eqn. 22

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### 2.11 Atmospheric Refraction Correction to the Solar

## Elevation ( $\Delta \mathrm{e}$ )

This correction factor is correlated as [9].
$\Delta \mathrm{e}=\frac{0.08421 \mathrm{P}}{\left[(273+\mathrm{T}) \tan \left(\frac{\mathrm{e}_{\mathrm{O}}+0.0031376}{\mathrm{e}_{\mathrm{O}}+0.089186}\right)\right]}$
Eqn. 23
where T is Temperature at a particular time of the day and $P$ is the pressure at that same time.

### 2.12 Local topocentric sun coordinates

### 2.12.1 Topocentric Zenith angle ( $\theta_{z}$ )

The solar zenith angle $\theta_{z}$ is the angle between a solar ray and local vertical direction and it is the complement of the altitude $A$ as illustrated in figure 2.
$\theta_{z}=\frac{\pi}{2}-e_{o}-\Delta e$
Eqn. 24

### 2.12.2 Topocentric Azimuth angle (A)

The solar azimuth is the angle between the line under the sun and the local meridian pointing to the equator, or due south in the northern hemisphere. It is positive when measured to the east and negative when measured to the west as illustrated in figure 2.
$\mathrm{A}=a \tan 2\left(\mathrm{sh}_{\mathrm{t}} \mathrm{ch}_{\mathrm{t}} \sin \phi-\tan \delta_{\mathrm{t}} \cos \phi\right)$
Eqn. 25

### 2.12.3 Topocentric Altitude angle ( $\alpha$ )

The solar altitude at a point on the earth is the angle between the line passing through the point and the sun and the line passing through the point tangent to the earth and passing below the sun as shown in figure 2 .
$\alpha=\sin \phi \sin \delta_{\mathrm{t}}+\cos \phi \cos \delta_{\mathrm{t}} \cosh _{\mathrm{t}}$
Eqn. 26
where; $\phi$ is the local latitude.

### 2.12.4 Solar incidence angle

In the design of solar energy systems, it is most important to be able to predict the angle between the sun's rays and a vector normal to the aperture or surface of the collector. This angle is called the angle of incidence $\theta_{i}$; the
maximum amount of solar radiation reaching the surface (or geometric aperture) is reduced by cosine of this angle. The other angle of importance is the tracking angle $\rho_{t}$. Most type of medium and high-temperature collectors require a tracking drive system to align at least one and often both axes of the collector aperture perpendicular to the sun's central ray. This is illustrated in figure 5.


Fig. 5. Solar Parabolic-trough Tracking Aperture where tracking rotation is about the axis.

For practical application when the tracking axis is oriented in the north-south direction the incidence angle, $\theta_{i}$, and the tracking angle, $\rho_{t}$, and are given by equation 27 and 28 respectively [10].
$\cos \theta_{i}=\sqrt{1-\cos ^{2} \alpha_{a} \cos ^{2} A}$
Eqn. 27
$\tan \rho_{t}=\frac{\sin A}{\tan \alpha_{t}}$

## Heat Gain, Heat Transfer and Heat Absorbed by the PTC

The solar radiation incident on the concentrator is reflected on to the receiver (absorber tube) located at the focal line through which working fluid flows and which is covered by concentric-transparent glass cover. As the temperature of the receiver increases, heat transfer processes take place. Energy in transition under the motive force of a temperature difference between components of the collector forms the basis for the determination of heat gained and heat losses to and from one component to
another. The PTC (Model TE38) used for the simulation shown in figure 6 has the following data:

| > | Inclination and azimuth | - | Adjustable |
| :---: | :---: | :---: | :---: |
| > | Parabolic reflector | - | Stainless Steel |
| $>$ | Aperture width ( W ) |  | 800mm (0.8m) |
| $>$ | Collector length ( L ) | - | 300 mm (0.3m) |
| $>$ | Focal length ( f ) | - | $487 \mathrm{~mm}(0.487 \mathrm{~m})$ |
|  | Aperture area ( $\mathrm{A}_{\mathrm{a}}$ ) | - | $0.24 \mathrm{~m}^{2}$ |
|  | Rim angle ( $\Psi_{\text {rim }}$ ) | - | $45^{0}$ |



Fig. 6. Basic Components of the Parabolic-Trough Collector

Energy balance equations for a PTC by by Egbo et al, [11], considered the heat-energy-gain, the heat-energyloss and the heat-energy-transfer between the components, i.e. the reflecting surface, the glass-cover and the absorber-tube, the thermal properties of the materials of the components and geometric dimensions of the SPTC. The equation for the enveloping glass-cover temperature developed by by Egbo et al, [11] is given as follows;

### 2.13.1 Energy Equations for the Enveloping Glass-Cover

The energy balance equation for the enveloping glasscover can be written as follows;

$$
2 \alpha_{g} R L\left[\left(I_{\text {beam }} * R_{b}\right)+I_{\text {diff }}\right]+
$$

$$
\begin{aligned}
& \rho_{c} \alpha_{g}\left[\frac{(W-D) L}{\pi}\right]\left(I_{\text {beam }} * R_{b}\right)+\frac{A_{t} \sigma\left(T_{t}^{4}-T_{g}^{4}\right)}{\frac{1}{\varepsilon_{t}}+\frac{A_{t}}{A_{g}}\left[\frac{1}{\varepsilon_{g}}-1\right]} \\
& -\sigma \varepsilon_{g} A_{g}\left(T_{g}^{4}-T_{s k y}^{4}\right)-A_{g} h_{c}\left(T_{g}-T_{\text {sur }}\right)= \\
& m_{g} c_{p_{g}} d T_{g} / d t
\end{aligned}
$$

Where: absorptance of the glass-cover material $\left(\alpha_{g}\right)$, outer radius of the enveloping glass-cover ( $R$ ),outer diameter of the enveloping glass-cover ( $D$ ), length of the concentrator ( $L$ ), tilt factor $\left(R_{b}\right)$, beam radiation on a horizontal surface ( $I_{\text {beam }}$ ), diffuse radiation ( $I_{\text {diff }}$ ), reflectance of the concentrator ( $\rho_{c}$ ), aperture width of the concentrator ( $W$ ), surface area of the enveloping glass-cover $\left(A_{g}\right)$, mass of the enveloping-glass-cover material ( $m_{g}$ ), specific heat capacity of the enveloping-glass-cover material $\left(C p_{g}\right)$, surface area of the absorber-tube $\quad\left(A_{t}\right)$, Stefan-Boltzmann's constant of radiation $(\sigma)$, temperature of the absorber-tube $\left(T_{t}\right)$, temperature of the enveloping glass-cover $\left(T_{g}\right)$, emittance of the enveloping glass-cover material ( $\varepsilon_{g}$ ), emittance of the coating material on the absorber-tube ( $\varepsilon_{t}$ ), sky temperature ( $\mathrm{T}_{\text {sky }}$ ), ambient temperature ( $\mathrm{T}_{\text {surr }}$ ) and convective heat transfer coefficient (h), temperature gradient of the enveloping-glass-cover ( $d T_{g} / d t$ ) as shown in figure 7. Where D is the outer diameter of the enveloping glass cover, $D_{1}$ is the inner
diameter of the enveloping glass cover, $\mathrm{X}_{\mathrm{g}}$ thickness of the enveloping glass cover.


Fig. 7: The enveloping glass-cover showing heat Gain and heat Loss

For purpose of solar concentrator design, it is often necessary to convert the beam radiation data on a horizontal surface to radiation on a tilt surface by using a conversion ratio $R_{b}$. This depends solely on the solar topocentric angles [11]. Consequently, the beam radiation incident on the aperture of the concentrating collector becomes $I_{\text {beam }} * R_{b}$.

$$
R_{b}=\frac{\cos (\phi-\rho) \cos \delta_{t} \cosh _{t}+\sin (\phi-\rho) \sin \delta_{t}}{\cos \phi \cos \delta_{t} \cos h_{t}+\sin \phi \sin \delta_{t}}
$$

Eqn. 30

### 2.13.2 Heat Gained and Heat Lost by the Absorber-Tube

The energy balance for the absorber-tube is as shown in figure 8 and can be written as in equation 31 follows;


Fig. 8: Representation of heat gain and heat loss by absorbertube.

$$
\begin{align*}
& \alpha_{t} \tau_{g}\left(\frac{2 R L}{\pi}\right)\left[\left(I_{\text {beam }} * R_{b}\right)+I_{\text {diff }}\right]+ \\
& \alpha_{t} \tau_{g} \rho_{c}\left[\frac{[(W-D) L}{\pi^{2}}\right]\left[I_{\text {beam }} * R_{b}\right]-\frac{A_{t} \sigma\left(T_{t}^{4}-T_{g}^{4}\right)}{\frac{1}{\varepsilon_{t}}+\frac{A_{t}}{A_{g}}\left[\frac{1}{\varepsilon_{g}}-1\right]}- \\
& \frac{A_{\text {in }}\left(T_{t}-T_{f}\right)}{\left[\frac{1}{h_{f}}+\frac{A_{i n} \ln \left(\frac{r}{r_{i}}\right)}{2 \pi K L}\right]}=m_{t} c_{p_{t}} d T_{t} / d t r \tag{Eqn. 31}
\end{align*}
$$

Where: absorptance of the absorber-tube material ( $\alpha_{t}$ ), transmittance of the enveloping glass-cover material ( $\tau_{g}$ ), surface area of the tube based on internal diameter of absorber-tube ( $A_{\text {tin }}$ ) and temperature of the fluid ( $T_{f}$ ), convective heat transfer coefficient of the fluid ( $h_{f}$ ), inner radius of the absorber-tube ( $r_{1}$ ) and outer radius of the absorber-tube ( $r$ ), mass of the absorbertube material $\left(m_{t}\right)$, specific heat capacity of the enveloping-glass-cover material ( $C p_{t}$ ) and temperature gradient of the absorber-tube ( $d T_{t} / d t$ ).

### 2.13.3. Energy Equation for the Working Fluid

The equation for the fluid temperature developed by Egbo et al, [11] is given as follows;
$\frac{A_{\text {in }}\left(T_{t}-T_{f}\right)}{\left[\frac{1}{h_{f}}+\frac{A_{\text {in }} \ln \left(\frac{r}{r_{1}}\right)}{2 \pi K L}\right]}-\frac{2 A_{c}\left(T_{f}-T_{\text {surr }}\right)}{\frac{1}{h_{c}}+\frac{1}{h_{f}}}=m_{f} c_{p_{f}} d T_{f} / d t \quad$ Eqn. 32
Where: mass of the working fluid ( $m_{f}$ ), specific heat capacity of the working fluid ( $C p_{f}$ ) and temperature gradient of the working fluid, $\left(\frac{d T_{f}}{d t}\right)$

### 2.13.4. The Thermal Efficiency of the Parabolic-Trough

Collector ( $\boldsymbol{\eta}_{\text {th }}$ )
The thermal efficiency of the Parabolic-trough
Collector is given by;

$$
\begin{equation*}
\eta_{t h}=1-\frac{Q_{\text {losses }}}{Q_{\text {input }}} \tag{Eqn. 33}
\end{equation*}
$$

where $\mathrm{Q}_{\text {losses }}$ is the total heat losses and $\mathrm{Q}_{\text {input }}$ is the total heat supplied to the receiver.

The hourly heat supplied to the receiver ( $\mathrm{Q}_{\text {input }}$ ) can be computed as follows:

$$
\begin{equation*}
Q_{\text {input }}=\left[\left(I_{\text {beam }} * R_{b}\right)+I_{\text {diff }}\right][W L] \tag{Eqn. 34}
\end{equation*}
$$

The total heat losses in the system $\left(\mathrm{Q}_{\text {losses }}\right)$ is considered as the sum of the radiative heat-loss from the surface of the enveloping glass-cover to the surroundings $\left(\mathrm{q}_{\mathrm{r}_{2}}\right)$, the convective heat loss from the surface of the enveloping glass-cover to the surroundings ( $\mathrm{q}_{\mathrm{c}}$ ) and the conductive/convective heat loss from the fluid to the surroundings $\left(\mathrm{q}_{1}\right)$. This can be expressed as follows;

$$
\begin{equation*}
Q_{\text {losses }}=q_{r_{2}}+q_{c}+q_{1} \tag{Eqn. 35}
\end{equation*}
$$

### 4.0 Results and Discussion

Mat lab program was employed for the simulation. The prediction of temperature change of the glass-cover, the absorber-tube and the fluid with change in time, using simulation model are carried out aimed at ascertaining the performance of a solar parabolic-trough concentrating collector (Model TE 38). The results are presented in Tables 1 and Figures 9-12.

From the result shown in figure 9, the temperature of the glass-cover increases with increase in time to a maximum value of $51^{\circ} \mathrm{C}$ at $16: 00 \mathrm{hrs}$ and subsequently reduces gradually to a value of $43^{\circ} \mathrm{C}$ at $19: 00 \mathrm{hrs}$. This is as a result of decrease in the intensity of the solar radiation falling on the collector which reduces as the sun set is approached. The transmissivity of the glasscover and energy losses from the glass-cover to surroundings may result in making the temperature of the glass-cover to be low; the higher the transmissivity of the material the more the solar radiation being transmitted to the absorber-tube, thus less temperature would be experienced on the glass cover [12]. This may
increase the efficiency of the PTC as more temperature is being received by the absorber-tube. The decrease in temperature that occurred between 9:00hr-10:00hr is due to cloudy condition and increase in velocity experienced in morning hours thus, reduces the temperature momentarily.

Table 1: Simulated Hourly Temperatures of the Glass-Cover $\left(T_{g}\right)$, Absorber-Tube $\left(T_{t}\right)$, Working Fluid

|  | 9:00hrs | 10:00hrs | 11:00hrs | 12:00hrs | 13:00hrs | $\mathbf{1 4 : 0 0 h r s}$ | 15:00hrs | 16:00hrs | 17:00hrs | 18:00hrs | $\mathbf{1 9 : 0 0}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left({ }^{\circ} \mathrm{C}\right)$ | 35 | 34 | 37 | 41 | 45 | 48 | 50 | 51 | 51 | 48 | 43 |
| $\left({ }^{\mathbf{0}} \mathrm{C}\right)$ | 61 | 63 | 67 | 75 | 83 | 92 | 101 | 110 | 117 | 122 | 125 |
| $\left({ }^{\mathbf{0}} \mathrm{C}\right)$ | 39 | 51 | 57 | 63 | 69 | 77 | 85 | 94 | 103 | 110 | 117 |
| $(\%$ | 90 | 95 | 95 | 95 | 94 | 93 | 91 | 89 | 83 | 72 | 62 |

$\left(T_{f}\right)$ and Thermal Efficiency $\left(\eta_{t h}\right)$



Fig. 9: Relationship between the Temperatures of the Glass-Cover and time of the day


Fig. 10: Relationship between the Temperatures of the Absorber-tube and time of the day

Figure 10 shows the relationship of the temperature of the absorber-tube with time of the day. The temperature increases gradually with time from $61^{\circ} \mathrm{C}$ at $9: 00 \mathrm{hrs}$ to a maximum temperature of $125^{\circ} \mathrm{C}$ at 19:00hrs. This trend is as a result of increase in the amount of solar radiation from the collector through the glass-cover to the absorber-tube which increases with time as the sun rises. The high transmissivity of the glass-cover, high absorptivity of the absorber-tube (copper tube) and coating of the absorber-tube with emulsion black paint that has low emittance has contributed to the rate of incident solar radiation absorbed by the absorber-tube. Also, the space between the glass-cover tube and the absorber-tube was hermetically closed and evacuated; it eliminates
the convective heat loss and reduces the heat losses due to radiation, because the produced heat from the absorber-tube is trapped as stated by Egbo, [13]. These factors greatly influence the temperature of the absorber-tube to increase.

Figure 11 shows the relationship between the temperatures of the fluid with time. The fluid temperature increases from $39^{\circ} \mathrm{C}$ at $9: 00 \mathrm{hrs}$ to $117^{\circ} \mathrm{C}$ at 19:00hrs with increase in time. This is as a result of conductive and convective heat transfer from the absorber-tube to the fluid inside the tube which increases with time.


Fig. 11: Relationship between Temperatures of the fluid and time of the day


Fig. 12: Relationship between thermal efficiency and time of the day

When the temperature difference between the fluid and the contact surface causes density variation in the fluid, convective heat transfer takes place and thus heat is
being transfer from the absorber-tube to the fluid [12].

The rate at which heat is actively removed from the absorber-tube to the fluid determines the operating temperature of the collector. The copper tube which was used as the absorber-tube is coated with emulsion black paint that has low emittance thermal property, increases the absorptance of the incident solar irradiance and reduces simultaneously the reflectance, thus increases the amount of heat transferred to the fluid [12].

A maximum efficiency of $95 \%$ was attained at the 11:00hrs, subsequently the efficiency reduces to a minimum value of $62 \%$ at 19:00hrs as shown in Figure 22. The thermal efficiency of the PTC reduces as a result of decrease in the intensity of the solar radiation falling on the collector which reduces as the sun set and the rate of energy losses within the collector system. Thus a model equation for simulation of a Parabolictrough Collector System using solar coordinates of Bauchi is successfully developed and tested.

## Conclusion

The input data considered for the model equations are the Universal Time (UT), Delta T( $\Delta T$ ), Date (day n, months M , and years Y ), longitude $\gamma$ and latitude $\phi$ (in radians) of Bauchi. The data also include the geometric global coordinates and local topocentric sun coordinates. The thermal efficiency $\left(\eta_{t h}\right)$ of the Parabolic-trough collector, considered both the total energy supplied to the receiver $Q_{\text {input }}$ and the total energy losses $Q_{\text {losses }}$ in the system. The results show that the temperatures of the glass-cover, absorber-tube, and the working fluid increases with increase in time.

The thermal efficiency of the collector increases with increase in time to a maximum value of $95 \%$ at 11:00hrs. However, it reduces to a minimum value of $62 \%$ at 19:00hrs.

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